

A CONTROL APPROACH FOR ROBOTS WITH FLEXIBLE LINKS AND RIGID END-EFFECTORS

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ABSTRACT

Multiarm flexible robots with dexterous end-effectors are currently being considered in such tasks as satellite retrieval, servicing and repair where a two-phase problem can be identified: Phase I, robot positioning in space; Phase II, object retrieval. In this article we only consider some issues in Phase I regarding modelling and control strategies for a robotic system comprised of a long flexible arm and a rigid three-link end-effector. The control objective is to maintain the last (rigid) link stationary in space in the presence of an additive disturbance caused by the flexible energy in the first link after a positioning maneuver has been accomplished. In our formulation, several configuration strategies can be considered, and optimal decentralized servocompensators can be designed. Preliminary computer simulations are included for a simple proportional controller to illustrate the approach.

I. Introduction

One of the requirements of current and future robot manipulators is that they be lightweight to permit rapid operation at a minimum energy cost. A typical application involves the (perhaps rapid) slewing of the shuttle arm in a satellite holding and retrieval task. Numerous studies have been done to find adequate controllers for robots consisting of one or more flexible/rigid links, including linear and nonlinear feedback, adaptive techniques, feed forward control, servocompensators, and optimal control [1-6].

One robot configuration that has not received attention is that of a long, lightweight (flexible) first link, followed by a dexterous end-effector consisting of rigid links numbered 2 onwards. The problem considered is that of keeping the end-effector fixed in space in orientation after a slew operation, which has been accomplished using time-optimal or minimal energy constraints. Due to the inherent flexibility of link 1, its tip will vibrate causing disturbances that affect (rigid) links 2 through n . The question we are addressing is twofold: 1) Can joints 2 through n move to keep link n fixed? What are the kinematic configurations that can accomplish this vibration compensation?, and 2) What control strategy must be followed by joints 2 on, for this compensation to be effective?

In answering these questions, we may consider the last link fixed and look at the reverse problem for a fictitious rigid manipulator comprised of

links $n - 1$ through 2 (the last link is considered the base). Thus, this is now a problem of analyzing the space that the tip of the fictitious rigid manipulator can move, and matching it to the vibrations of a single link manipulator. A number of different solutions and configurations can be considered depending on the type of joints that the rigid manipulator has. The manipulator considered here is shown in Figure 1. The system consists of four links labeled L1 through L4. The first link is very long and flexible ($L1 \gg L2, L3, L4$), and the last three links represent a rigid robot arm with an end-effector. The hub joint is revolute to allow planar or single-axis slewing maneuvers; the second joint is also revolute, the third joint is both revolute and prismatic, and the fourth joint is revolute.

The following assumptions/simplifications are made:

- * The flexible disturbance is additive.
- * The end-effector robot can be modelled as a lumped point mass.

The modelling restriction may be relaxed in a later study and a more detailed model can be obtained [7-9].

Definition 1:

The rigid configuration is the orientation that the arm and end-effector would assume if the arm were rigid.

The main goal of this paper is to solve the control problem of system orientation about the rigid configuration in the presence of flexible disturbances. One particular configuration strategy is considered in the following section.

II. A Configuration Strategy

The problem formulated in the Introduction is now solved within the framework of a particular configuration strategy. To that end, consider the relegation of control effort to Joint J2 so that link L2 remains parallel to the rigid configuration. We will label this strategy as the absolute flexible-to-rigid configuration strategy. Given this orientation constraint, our task is to determine the appropriate control commands for joints J2, J3, J4.

Figure 1 illustrates the rigid configuration which is also the "steady-state" orientation after the flexible disturbance has disappeared. We attach the coordinate frames {1}, {2}, {3}, and {4} to each of the links as shown in the figure. Reference frame {0} is an inertial coordinate system. Using these frames, a point 1P_1 in {1}, for example, can be expressed in {2} as 2P_1 by the relation:

$$\begin{bmatrix} {}^2P_1 \\ 1 \end{bmatrix} = {}^2_1T \begin{bmatrix} {}^1P_1 \\ 1 \end{bmatrix}$$

where the homogeneous frame transformation 2_1T is given by

$${}^2_1T = \begin{bmatrix} {}^2_1R(\theta_{12}) & {}^2P_{1org} \\ 0 & 1 \end{bmatrix}; {}^2_1R(\theta_{12}) = \begin{bmatrix} \cos(\theta_{12}) & \sin(\theta_{12}) \\ -\sin(\theta_{12}) & \cos(\theta_{12}) \end{bmatrix}$$

where ${}^2P_{1org}$ is a vector that expresses the origin of {1} in frame {2}.

Assume that a suitable rigid configuration has been determined with the corresponding solution set

$$\cdot \left(\psi_{01}^r, \psi_{12}^r, (\psi_{23}^r, x^r), \psi_{34}^r \right)$$

where, as illustrated in Figure 1, ψ_{ij}^r is the angle between frame i and frame j, and x^r is the displacement of the prismatic joint.

We next define "flexible coordinate frames" {1F}, {2F}, and {3F} which coincide exactly with frames {1}, {2}, and {3}, respectively, once the flexible disturbance has disappeared. We also define the flexibility pair (β , Y_{TIP}) where β is a small flexibility angle measured locally at the tip of link L1 and Y_{TIP} is the corresponding local transverse displacement. Then,

$$\psi_{01} = \psi_{0,1F} = \psi_{01}^r + \beta$$

and for the configuration strategy under consideration,

$$\psi_{02} = \psi_{0,2F} = \psi_{02}^r$$

from which it follows that

$$\psi_{12} = \psi_{1,2F} = \psi_{12}^r - \beta. \quad (1)$$

To determine the flexible displacement $x = x^f$ of the prismatic joint, we observe that

$${}^3F_4P_{4org} = \begin{bmatrix} x^f \\ 0 \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix} = -{}^3F_4R \begin{bmatrix} p \\ q \end{bmatrix} \quad (2)$$

$$\text{where } \begin{bmatrix} p \\ q \end{bmatrix} = {}^4_3R {}^3_2R {}^2_1R \begin{bmatrix} L_1(\cos(\beta)-1) \\ Y_{TIP} \end{bmatrix} - {}^4_3R \begin{bmatrix} x^r \\ 0 \end{bmatrix}$$

$$\text{and } \psi_{43} = \psi_{4,3F} = \begin{cases} \text{ATAN}(\frac{q}{p}) & \text{if } p < 0 \\ \pi + \text{ATAN}(\frac{q}{p}) & \text{if } p > 0 \text{ and } q < 0 \\ -\pi + \text{ATAN}(\frac{q}{p}) & \text{if } p > 0 \text{ and } q > 0 \end{cases} \quad (3)$$

Finally, the angular set point of joint J3 is found to be

$$\Psi_{32} = \Psi_{3F,2F} = \Psi_{32}^r - [\Psi_{43} - \Psi_{43}^r] \quad (4)$$

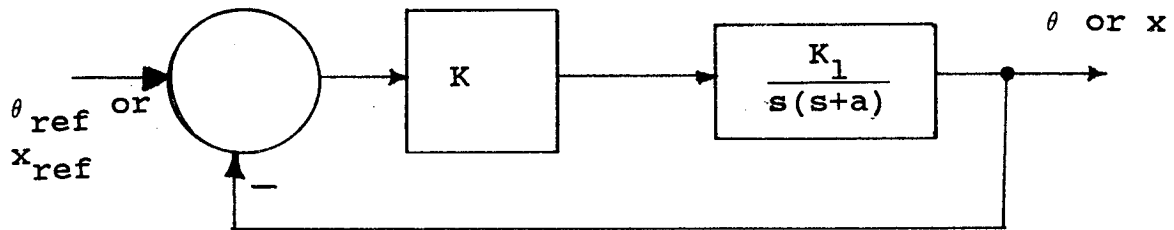
Equations 1-4 are the solution to the regulation problem for this particular configuration strategy. Note that the joint coordinates can be found from the following relations:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = {}^0_1T \begin{bmatrix} L_1 \\ 0 \\ 1 \end{bmatrix} ; \begin{bmatrix} x_3 \\ y_3 \\ 1 \end{bmatrix} = {}^0_1T {}^1_2T \begin{bmatrix} L_2 \\ 0 \\ 1 \end{bmatrix} \quad (5a)$$

$$\begin{bmatrix} x_4 \\ y_4 \\ 1 \end{bmatrix} = {}^0_1T {}^1_2T {}^2_3T \begin{bmatrix} x \\ 0 \\ 1 \end{bmatrix} ; \begin{bmatrix} x_5 \\ y_5 \\ 1 \end{bmatrix} = {}^0_1T {}^1_2T {}^2_3T {}^3_4T \begin{bmatrix} L_4 \\ 0 \\ 1 \end{bmatrix} \quad (5b)$$

III. Simulations

In this section we include some representative simulations that illustrate the results obtained thus far. For simplicity, we assume that all motors are identical and can be modelled as shown in Figure 2.



$$H(s) = \frac{\omega_o^2}{s^2 + 2 \xi \omega_o s + \omega_o^2}$$

Figure 2. Motor Model

where θ is the angle of a revolute joint and x is the linear displacement of a prismatic joint. The gain K can be chosen so that a suitable response is obtained. In the simulations we set the damping ratio $\xi = 0.707$ and ω_o is a design parameter. This simple proportional controller is used to illustrate the results and should be viewed only as a representative design. A procedure to construct a suitable finite dimensional model of the slewing beam can be found in [10]. A slewing profile can be generated by applying a torque input to the model with hub rate and angle feedback [11]. The physical parameters used, structural modes and closed-loop modes are listed in Table 1. We have chosen the first eight modes to constitute a "truth" model for the flexible

beam. A typical simulation of hub angle ψ_{01} and tip position Y_{TIP} is shown in Figure 3 for two and eight modes. Notice that no attempt is made to decrease the structural vibrations and a "poor" slewing profile is deliberately generated. Table 2 lists the initial and final rigid system configurations.

Equations 1-4 are used to compute the required motor set-points based on a reduced order model of the beam that retains two or four modes. The resulting joint angles and the true hub angle and tip position (from the 8-mode model) are then used to compute the position of the links (see Equation 5). One representative simulation is given in Figure 4. An expanded view of the error in the coordinates of the last link is shown in Figure 5. There, we observe that the y-coordinate of the tip (y_5) has a maximum error of about 2 cm. Table 3 compiles the results for several simulation runs and different choices of motor bandwidth and number of modes used in the reduced-order model of the flexible link. Increasing the motor's bandwidth reduces the overall regulation error to some extent. The major contribution to the error is seen to be the inexact account of the total deviation of link L1 from its rigid position. This is a consequence of employing a controller based on a reduced-order model of the flexible beam.

IV. Conclusions and Further Research

We have presented a control strategy for a flexible manipulator with rigid end-effector. Our preliminary results are promising, and current research efforts include analysis of other configuration strategies, extension to a similar robotic system with a three-dimensional workspace, coordination of multiple robotic arms, and implementation of an optimal decentralized servocompensator [11,12] for each motor for reference tracking and flexible disturbance rejection.

V. References

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Table 1. Physical Link Parameters and Modes

Link 1	$L_1 = 4.0\text{m}$; $\rho = 0.4847 \text{ Kg/m}$; $I = 3.3339 \times 10^{-11}\text{m}^4$ $E = 6.8944 \times 10^9 \text{ N/m}^2$; $A = 1.5875 \times 10^{-4}\text{m}^2$ Tip Mass = 0.2 Kg $L_2 = L_3 = 0.5 \text{ m}$; $L_4 = 0.2 \text{ m}$
Structural Modes (Hz)	0 ; 0.0884 ; 0.2635 ; 0.4828 ; 0.8151 1.3090 ; 1.9535 ; 2.7426 ; 3.6846
Closed Loop Modes	$-1.7723 \times 10^{-4} \pm j \ 0.1177$; $-6.4417 \times 10^{-3} \pm j \ 0.7731$ $-6.2638 \times 10^{-2} \pm j \ 2.2021$; $-5.1936 \times 10^{-1} \pm j \ 3.9712$ $-3.6426 \times 10^{-1} \pm j \ 5.2491$; $-7.4596 \times 10^{-2} \pm j \ 8.2481$ $-2.6528 \times 10^{-2} \pm j \ 12.281$; $-1.1691 \times 10^{-2} \pm j \ 17.234$ $-7.4112 \times 10^{-3} \pm j \ 23.152$

Table 2. Initial and Final Configurations
(Ψ_{ij} in degrees ; x,y in meters)

Ψ_{01}	Ψ_{12}	Ψ_{23}	x	Ψ_{34}	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
55	45	-80	0.2	-30	2.29	3.27	2.21	3.77	2.39	3.84	2.59	3.81
25	20	-40	0.1	-20	3.63	1.69	3.98	2.04	4.08	2.05	4.27	2.00

Table 3. Error Range in Link 4 Displacement

- * Time Interval = 15-30 sec
- * First entry corresponds to 2 mode controller
- * Second entry corresponds to 4 mode controller

BW (Hz)	Error Range (cm)			
	x_4	y_4	x_5	y_5
1	± 3 ; 0,3.5	± 6 ; 0,7	± 3 ; 0,3.5	± 6 ; 0,7
2	± 2.5 ; 0,2	± 4.5 ; 0,4	± 2.5 ; 0,1.9	± 5 ; 0,4
5	-3,2 ; 0,1.2	-4,6 ; 0,2.25	-3,2 ; 0,1.1	-4,6 ; 0,2.4
10	-3,2 ; 0,1	-4,6 ; 0,2	-3,2 ; 0,0.9	-4,6 ; 0,2.1
30	-3,2 ; 0,1	-4,6 ; 0,2	-3,2 ; 0,0.9	-4,6 ; 0,2.1

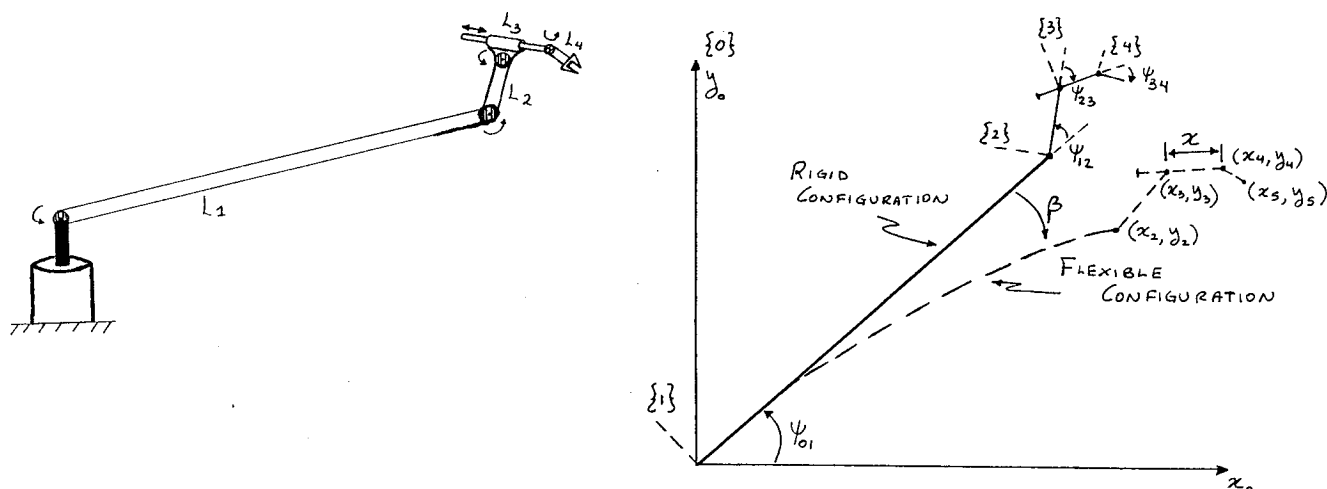


Figure 1. The Flexible Manipulator.

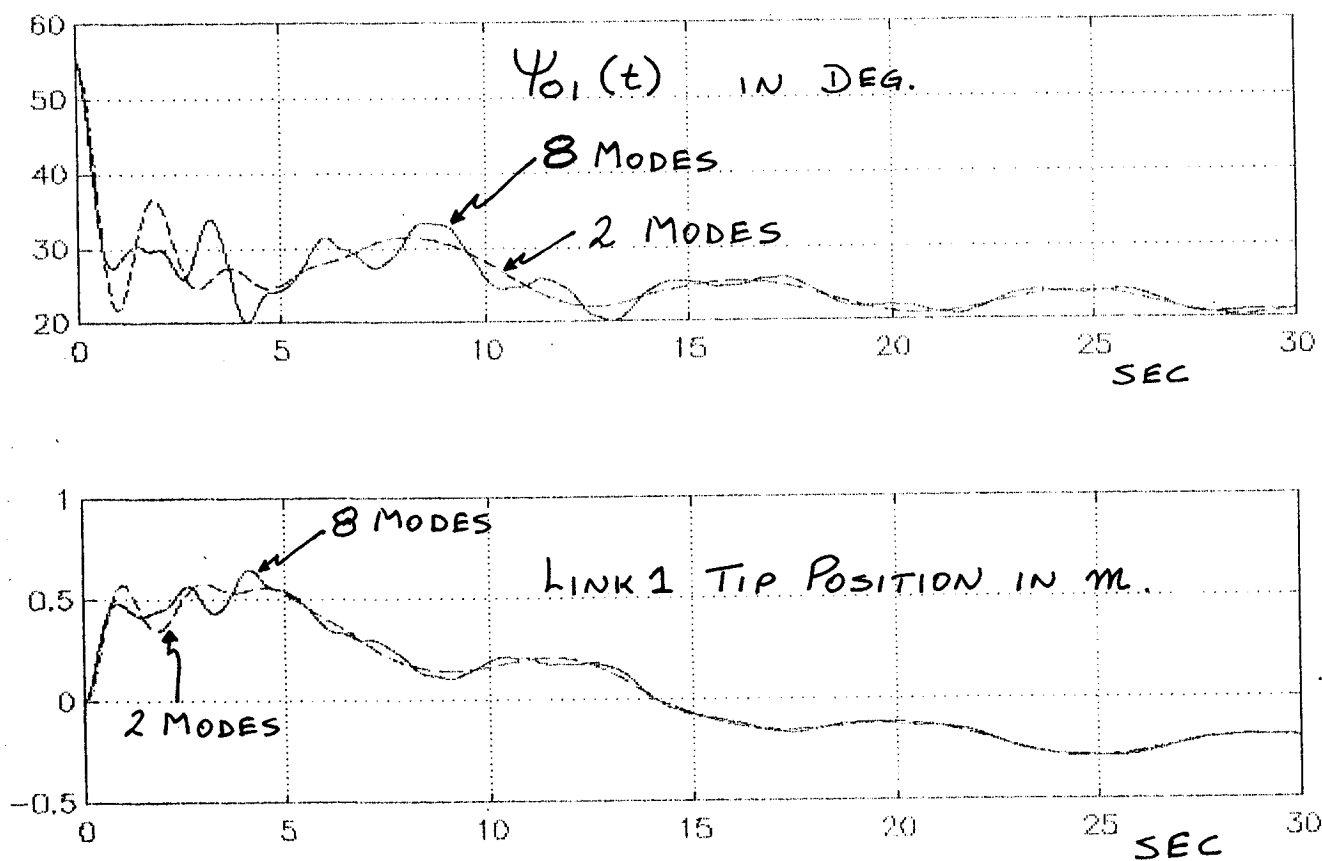


Figure 3. Hub Angle and Local Tip Position of Link 1.

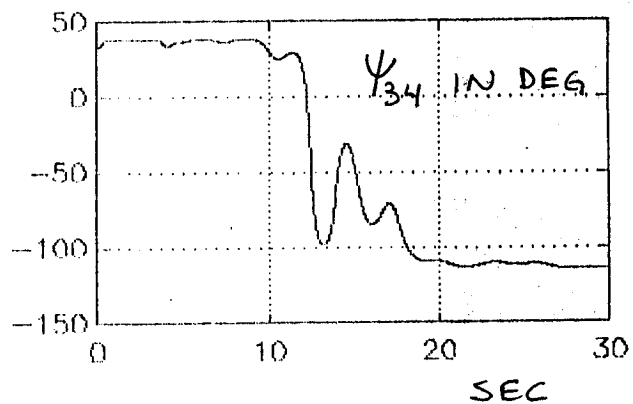
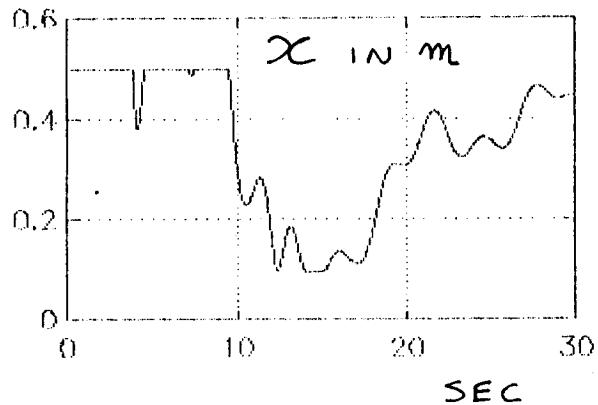
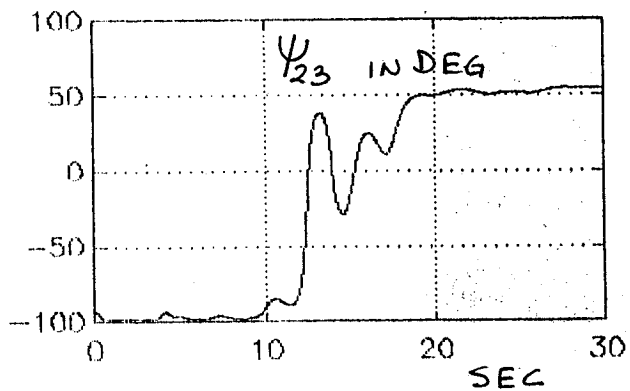
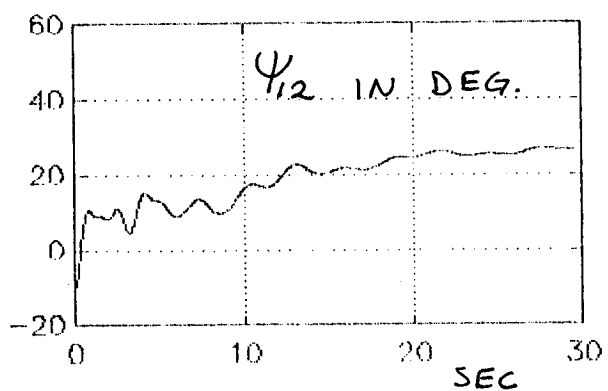


Figure 4. Joint Angles and Prismatic Displacement for a 4 mode Controller and 30 Hz Bandwidth.

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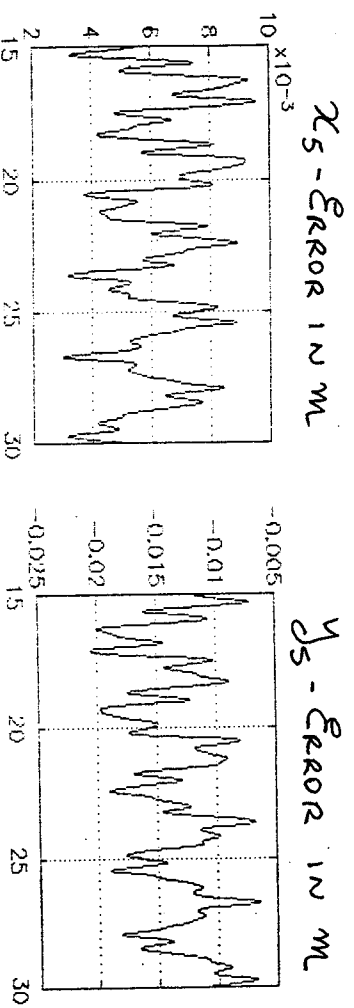
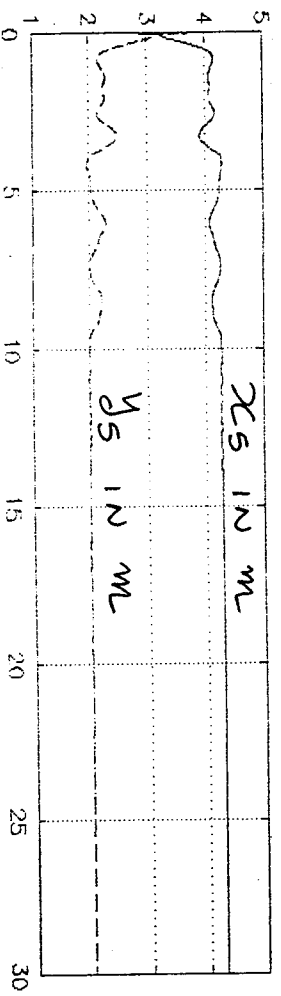
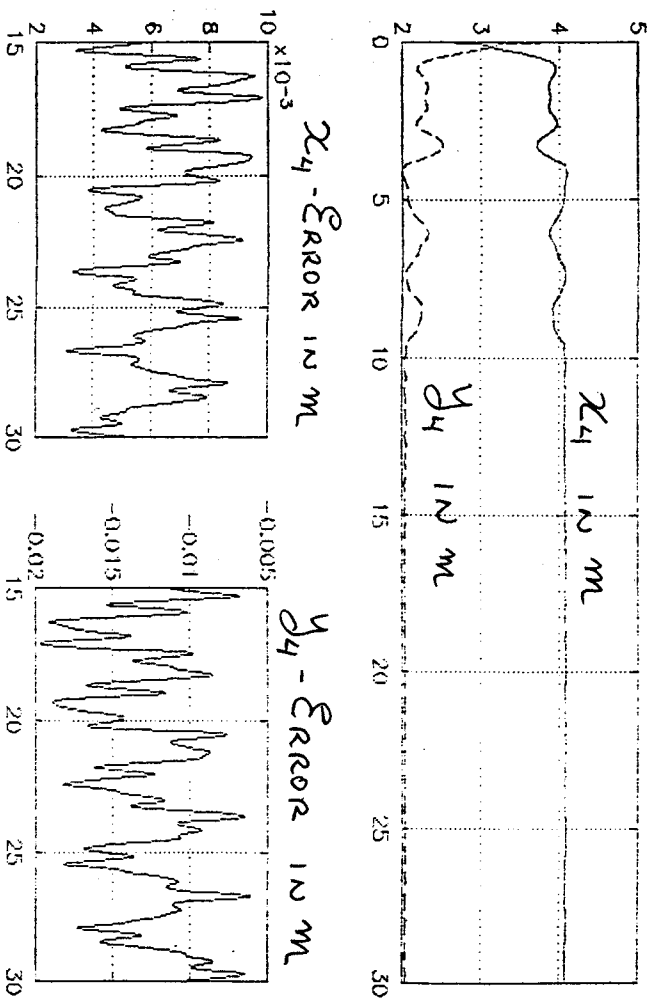


Figure 5. Coordinates of Link 4 and Expanded View of Errors.